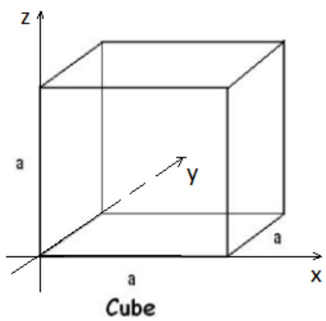
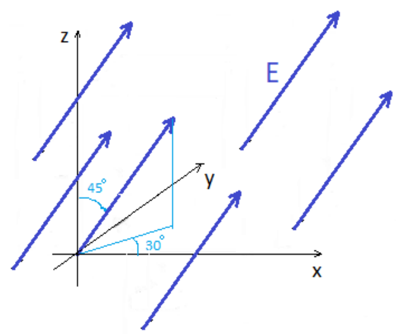
Homework 2 *(Solutions)*

* Some of the problems below require calculating integrals and solving transcendental equations. This not being a math class per se´, all I care about is seeing the integral you're trying to antidifferentiate, and seeing the equation you're trying to solve. But as far as the actual *calculation* of the antiderivative or solution to the equation, as far as I'm concerned, you may use calculator/math software/wolfram alpha, etc.

Problem 1. An empty cube (a = 80cm) floating around the internet, is immersed in a constant electric field **E** = 90N/C which makes a $\phi = 30°$ angle with the +x axis, and a $\theta = 45°$ with the +z-axis.





(a) Should the electric flux through the front surface be positive or negative? What is the area vector of the front surface? What is the flux through the front surface?

Should be negative as field lines are going *in*.

$A_{front} = a^2(-\mathbf{j}) = (0.80m)^2(-\mathbf{j}) = -0.64m^2\mathbf{j}$.

Flux = $\mathbf{E}\cdot\mathbf{A} = (90\cos45°\mathbf{k} + \sin45°\cos30°\mathbf{i} + \sin45°\sin30°\mathbf{j})\cdot(-0.64\mathbf{j}) = (64\mathbf{k} + 55\mathbf{i} + 32\mathbf{j})\cdot(-0.64\mathbf{j}) = -20.5Nm^2/C$

(b) Should the electric flux through the back surface be positive or negative? What is the area vector of the back surface? What is the flux through the back surface?

Should be positive as field lines are going *out*.

$A_{back} = a^2(\mathbf{j}) = (0.80m)^2(\mathbf{j}) = 0.64m^2\mathbf{j}$.

Flux = $\mathbf{E}\cdot\mathbf{A} = (64\mathbf{k} + 55\mathbf{i} + 32\mathbf{j})\cdot(0.64\mathbf{j}) = 20.5Nm^2/C$

(c) Should the electric flux through the left side surface be positive or negative? What is the area vector of the left side surface? What is the flux through the left side surface?

Should be negative as field lines are going *in*.

$A_{leftside} = a^2(-\mathbf{i}) = (0.80m)^2(-\mathbf{i}) = -0.64m^2\mathbf{i}$.

Flux = $\mathbf{E}\cdot\mathbf{A} = (64\mathbf{k} + 55\mathbf{i} + 32\mathbf{j})\cdot(-0.64\mathbf{i}) = -35.2Nm^2/C$

(d) Should the electric flux through the right side surface be positive or negative? What is the area vector of the right side surface? What is the flux through the right side surface?

Should be positive as field lines are going *out*.

$A_{rightside} = a^2(\mathbf{i}) = (0.80m)^2(\mathbf{i}) = 0.64m^2\mathbf{i}$.

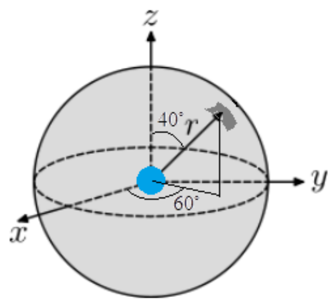
Flux = $\mathbf{E}\cdot\mathbf{A} = (64\mathbf{k} + 55\mathbf{i} + 32\mathbf{j})\cdot(0.64\mathbf{i}) = 35.2Nm^2/C$

(e) Should the electric flux through the top surface be positive or negative? What is the area vector of the top surface? What is the flux through the top surface?
Should be positive as field lines are going *out*.
$A_{top} = a^2(\mathbf{k}) = (0.80\text{m})^2(\mathbf{k}) = 0.64\text{m}^2\mathbf{k}$.
Flux = $\mathbf{E}\cdot\mathbf{A} = (64\mathbf{k} + 55\mathbf{i} + 32\mathbf{j})\cdot(0.64\mathbf{k}) = 41\text{Nm}^2/\text{C}$

(f) Should the electric flux through the bottom surface be positive or negative? What is the area vector of the bottom surface? What is the flux through the bottom surface?
Should be negative as field lines are going *in*.
$A_{top} = a^2(\mathbf{k}) = (0.80\text{m})^2(-\mathbf{k}) = -0.64\text{m}^2\mathbf{k}$.
Flux = $\mathbf{E}\cdot\mathbf{A} = (64\mathbf{k} + 55\mathbf{i} + 32\mathbf{j})\cdot(-0.64\mathbf{k}) = -41\text{Nm}^2/\text{C}$

(g) What is the total flux through the cube? Is this consistent with Gauss's law?
Total flux is 0, as Gauss's law implies, since $q_{enclosed} = 0$.

Problem 2. Now consider a -2μC charge sitting at the origin, and spherical surfaces of radius r = 35cm surrounding it.



(a) Consider the small grey(er) patch of area on the sphere's surface. Say it's about 1cm across and 2cm long.

i) What is it's area vector d**A**?
This is: d**A** = dA(*away from origin*) = (1cm×2cm)[cos40°**k** + sin40°cos60°**i** + sin40°sin60°**j**) = (0.01m×0.02m)(0.77**k** + 0.32**i** + 0.56**j**) = $(1.54\times10^{-4}\mathbf{k} + 0.64\times10^{-4}\mathbf{i} + 1.12\times10^{-4}\mathbf{j})$ (m^2)

ii) What is the electric field vector **E** passing through that patch?
Yeah, well,

$$\begin{aligned}
\mathbf{E} &= \frac{k|q|}{r^2} \text{ (towards origin)} \\
&= \frac{(9\times10^9)(2\times10^{-6})}{0.35^2}\left[-0.77\hat{\mathbf{k}} - 0.32\hat{\mathbf{i}} - 0.56\hat{\mathbf{j}}\right] \\
&= 1.47\times10^5 \text{ N/C}\left[-0.77\hat{\mathbf{k}} - 0.32\hat{\mathbf{i}} - 0.56\hat{\mathbf{j}}\right] \\
&= -1.13\times10^5\hat{\mathbf{k}} - 0.47\times10^5\hat{\mathbf{i}} - 0.82\times10^5\hat{\mathbf{j}}
\end{aligned}$$

iii) What is the electric flux **E**·d**A** passing through that patch?

And that's,

$$
\begin{aligned}
d\Phi &= \mathbf{E}\cdot d\mathbf{A} \\
&= (-1.13\times10^5\hat{\mathbf{k}} - 0.47\times10^5\hat{\mathbf{i}} - 0.82\times10^5\hat{\mathbf{j}})\cdot(1.54\times10^{-4}\hat{\mathbf{k}} + 0.64\times10^{-4}\hat{\mathbf{i}} + 1.12\times10^{-4}\hat{\mathbf{j}}) \\
&= -(1.13\times10^5\cdot1.54\times10^{-4}) - (0.47\times10^5\cdot0.64\times10^{-4}) - (0.82\times10^5\cdot1.12\times10^{-4}) \\
&= -29.6\ \mathrm{Nm^2/C}
\end{aligned}
$$

(b) If we took that same patch but changed its position on the sphere (by changing ϕ, or θ),

i) would dA change? Nope.

ii) Would d**A** change? Yep, because direction is different.

iii) Would E change? Nope.

iv) Would **E** change? Yeah, 'cause direction is different.

v) Would the flux change? Nope, since **E** and d**A** would still be pointing directly opposite direction, same as before.

(c) How many such patches would fit into the entire surface? What would therefore be the flux passing through the entire surface? Is this consistent with Gauss's law?
Number of patches is: $N = A_{sphere}/A_{patch} = 4\pi(0.35\mathrm{m})^2/(0.01\mathrm{m}\times0.02\mathrm{m}) = 7700$.
$\Phi = Nd\Phi = (7700)(-29.6\ \mathrm{Nm^2/C}) = -2.28\times10^5\ \mathrm{Nm^2/C}$
Gauss's law says Φ ought to be $q_{enclosed}/\varepsilon_0 = (-2\times10^{-6})/(8.85\times10^{-12}) = -2.26\times10^5\ \mathrm{Nm^2/C}$. So yes it agrees.

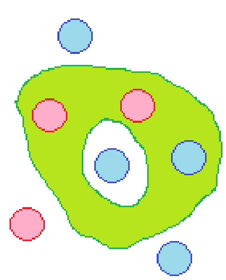
(d) Would the flux through the entire surface change if we:

i) decreased the radius? Nope.

ii) increased the radius? Nope.

iii) turned it into another shape, like a box? Nope.

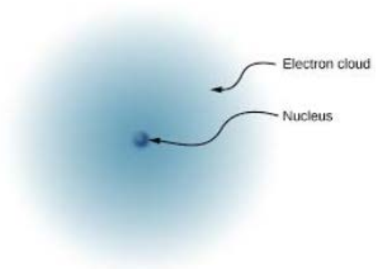
Problem 3. Each of the pink charges is +1µC, and each of the blue charges is -1µC. What is the total flux passing through the green closed surface?



Well that's easy. Just, $\Phi = q_{enclosed}/\varepsilon_0 = (+1\mu C + 1\mu C - 1\mu C)/\varepsilon_0 = 1.13\times10^5\ Nm^2/C$.

Problem 4. According to the classical picture of a Hydrogen atom, it consists of a proton nucleus orbited by an electron. But in reality the electron doesn't *orbit* at all, it *hovers* around the nucleus as a diffuse 'cloud'. Due to the proton's attractive force, the electron cloud is most dense near the proton. Using quantum mechanics, we can precisely calculate the electron cloud's properties. And we would find that the cloud's charge density is given by the equation below (r = distance from proton).

$$\rho_e(r) = \frac{-e}{\pi a^3} e^{-2r/a} \left(\frac{\mathrm{C}}{\mathrm{m}^3}\right) \qquad a = \text{Bohr radius} = 53\,\text{pm}$$



(a) What radius encloses 90% of the electron (i.e. 90% of the electron's charge)?
So we want,

$$0.90(-e) = \int_0^r \rho_e(r)dV$$

$$0.90(-e) = \int_0^r \frac{-e}{\pi a^3} e^{-2r/a} \cdot 4\pi r^2 dr$$

$$0.90 = \int_0^r \frac{4r^2}{a^3} e^{-2r/a} dr$$

$$0.90 = 1 - \frac{e^{-\frac{2r}{a}}}{a^2}(a^2 + 2ar + 2r^2)$$

$$\frac{e^{-\frac{2r}{a}}}{a^2}(a^2 + 2ar + 2r^2) = 0.10$$

$$e^{-2\left(\frac{r}{a}\right)}\left[1 + 2\left(\frac{r}{a}\right) + 2\left(\frac{r}{a}\right)^2\right] = 0.10$$

Now we must solve this equation. Here's a tricky thing. While not necessary, we can replace r/a with the variable 'u'. That makes the equation easier to handle,

$$e^{-2u}\left[1 + 2u + 2u^2\right] = 0.10 \qquad u = r/a$$

If you plug this equation into Wolfram, then you get the solution u = 2.66 ⟶ r = 2.66a = 2.66(53pm) = 141pm.

(b) What is the electric field at this radius? Is it pointing in or out? (don't forget about the nucleus)
Charge enclosed is $q_{enclosed} = q_{nucleus} + q_{electron\ cloud}$ = (e) + (-0.90e) = 0.10e. So field strength is:

$$E = \frac{kq_{enclosed}}{r^2} = \frac{(9\times10^9)(0.10e)}{(141\times10^{-12})^2} = 7.2\times10^9\text{ N/C} \qquad \text{(outwards)}$$

(c) Give a general symbolic expression for the electric field at radius r.
From Gauss's law:

$$\begin{aligned}E &= \frac{kq_{enclosed}}{r^2}\\ &= \frac{k}{r^2}\left[q_{nucleus} + q_{cloud}\right]\\ &= \frac{k}{r^2}\left[e + \int_0^r \rho_e(r)dV\right]\\ &= \frac{k}{r^2}\left[e - \int_0^r \frac{e}{\pi a^3}e^{-2r/a}\cdot 4\pi r^2 dr\right]\\ &= \frac{k}{r^2}\left[e - e\left[\frac{e^{-\frac{2r}{a}}}{a^2}(a^2+2ar+2r^2)-1\right]\right]\\ &= \frac{ke}{r^2}\left[2 - e^{-\frac{2r}{a}}\left(1+2\frac{r}{a}+2\frac{r^2}{a^2}\right)\right]\end{aligned}$$

Problem 5. Manufacturing facilities can generate large, charged, wood or metallic dust clouds via all the sawing, shaving, boring, etc., that goes on (the dust gets charged by friction, similar to the process you used in the lab). If the cloud gets large/dense enough, the electric field it generates will become strong enough to start stripping the electrons off of the surrounding air molecules (this is called dielectric breakdown and is what happens when you get lightening). The resulting cascade of electrons would rapidly collide with all the air/dust molecules, delivering a ton of kinetic energy to the molecules very quickly, thereby causing their temperature to rise very quickly, thereby causing them to expand very quickly – i.e., an explosion. Such explosions have completely demolished factories.

So….suppose our factory's manufactoring process creates spherical aluminum ($\rho_{A\ell}$ = 2700kg/m^3) dust particles 100µm in diameter, each charged with 1pC. And say that the dust cloud's density is uniformly ρ_{cloud} = 0.05kg/m^3 (this is much less than air's density, and so ought to be small enough to float).

Aftermath of 2008 explosion at Imperial Sugar in Port Wentworth, Georgia, US

(a) Say the dustcloud has a radius R = 3m. Derive an expression for the electric field E, as a function of r (r being distance from center of the cloud). And plot it below:

The electric field of the cloud would be given by:

$$E = \frac{kq_{enclosed}}{r^2}$$

Inside the cloud, we'd have:

$$\begin{aligned}
q_{enclosed} &= q_{particle} \frac{\#\, particles}{volume} \cdot volume \\
&= q_{particle} \frac{mass\ particles}{volume} \frac{\#\, particles}{mass\ particles} \cdot volume \\
&= (1\times10^{-12}\,\mathrm{C})(0.05\,\mathrm{kg/m^3}) \frac{1}{\frac{4}{3}\pi(50\times10^{-6}\,\mathrm{m})^3(2700\,\mathrm{kg/m^3})} \cdot \frac{4}{3}\pi r^3 \\
&= 1.48\times10^{-4} r^3\ (\mathrm{C})
\end{aligned}$$

Outside the cloud we'd have:

$$\begin{aligned}
q_{enclosed} &= 1.48\times10^{-4} \cdot (3)^3\ (\mathrm{C}) \\
&= 0.004\,(\mathrm{C})
\end{aligned}$$

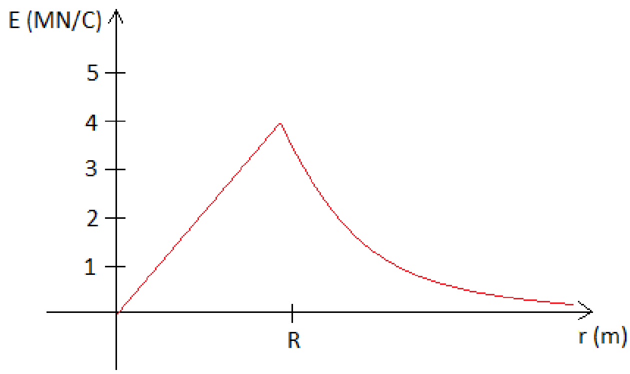
And so the field would be given by:

$$E = \frac{kq_{enclosed}}{r^2}$$

$$= \begin{cases} \dfrac{9\times10^9 \cdot (1.48\times10^{-4} r^3)}{r^2} & \text{inside} \\ \dfrac{9\times10^9 (0.004)}{r^2} & \text{outside} \end{cases}$$

$$= \begin{cases} 1.33\times10^6 r & \text{inside} \\ \dfrac{36\times10^6}{r^2} & \text{outside} \end{cases}$$

Rough plot looks like this:



(b) What radius dust cloud R would suffice to create an E = 3MN/C electric field, able to initiate dielectric breakdown of the air?

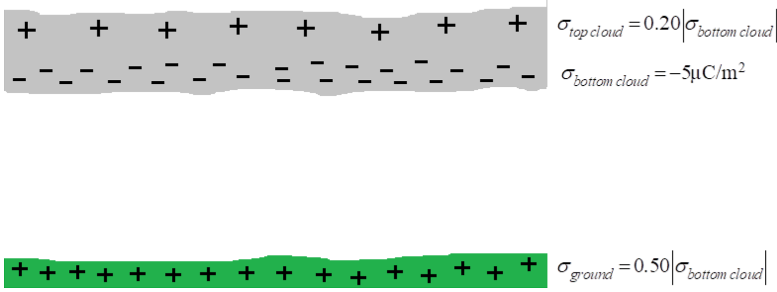
This would be,

$$E_{\max} = 3\times10^6$$

$$(1.33\times10^6)(R) = 3\times10^6$$

$$R = \frac{3\times10^6}{1.33\times10^6} = 2.26\,\text{m}$$

Problem 6. Speaking of dielectric breakdown…when water evaporates, it rises, and as it ascends to higher elevations (and colder temperatures), will cool into powdery icey slosh and descend. The collisions between ascending water vapor and descending icey slosh will negatively charge the icey slosh, and consequently positively charge the ascending vapor, much like you charged objects via contact in the lab. These charged aggregates, still fine enough to float will form the contents of thunderstorm cloud. Convection currents will disperse a large portion of the positively charged topside of the cloud, and negatively charged bottomside of the cloud will partially charge the ground, by induction (same induction as you learned in lab). Let's say we have the following situation then,



(a) What is the magnitude and direction of the electric field in the air between the cloud and the ground?

Well,

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}_{top\ cloud} + \mathbf{E}_{bottom\ cloud} + \mathbf{E}_{ground} \\
&= \frac{|\sigma_{top\ cloud}|}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{|\sigma_{bottom\ cloud}|}{2\varepsilon_0}(\hat{\mathbf{j}}) + \frac{|\sigma_{ground}|}{2\varepsilon_0}(\hat{\mathbf{j}}) \\
&= \frac{(0.2)(5\times10^{-6})}{2(8.85\times10^{-12})}(-\hat{\mathbf{j}}) + \frac{5\times10^{-6}}{2(8.85\times10^{-12})}(\hat{\mathbf{j}}) + \frac{(0.5)(5\times10^{-6})}{2(8.85\times10^{-12})}(\hat{\mathbf{j}}) \\
&= [-0.2+1+0.5]\frac{(5\times10^{-6})}{2(8.85\times10^{-12})}\hat{\mathbf{j}} \\
&= 3.6\times10^{5}\text{ N/C}\,\hat{\mathbf{j}}
\end{aligned}$$

(b) What is the magnitude and direction of the electric field in between the two charged regions of the cloud?

This is:

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}_{top\ cloud} + \mathbf{E}_{bottom\ cloud} + \mathbf{E}_{ground} \\
&= \frac{|\sigma_{top\ cloud}|}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{|\sigma_{bottom\ cloud}|}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{|\sigma_{ground}|}{2\varepsilon_0}(\hat{\mathbf{j}}) \\
&= \frac{(0.2)(5\times10^{-6})}{2(8.85\times10^{-12})}(-\hat{\mathbf{j}}) + \frac{5\times10^{-6}}{2(8.85\times10^{-12})}(-\hat{\mathbf{j}}) + \frac{(0.5)(5\times10^{-6})}{2(8.85\times10^{-12})}(\hat{\mathbf{j}}) \\
&= [-0.2-1+0.5]\frac{(5\times10^{-6})}{2(8.85\times10^{-12})}\hat{\mathbf{j}} \\
&= 2\times10^{5}\text{ N/C}
\end{aligned}$$

(c) What is the magnitude and direction of the electric field above the cloud?

This would be:

$$\mathbf{E} = \mathbf{E}_{top\ cloud} + \mathbf{E}_{bottom\ cloud} + \mathbf{E}_{ground}$$

$$= \frac{\left|\sigma_{top\ cloud}\right|}{2\varepsilon_0}(\hat{\mathbf{j}}) + \frac{\left|\sigma_{bottom\ cloud}\right|}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{\left|\sigma_{ground}\right|}{2\varepsilon_0}(\hat{\mathbf{j}})$$

$$= \frac{(0.2)(5\times10^{-6})}{2(8.85\times10^{-12})}(\hat{\mathbf{j}}) + \frac{5\times10^{-6}}{2(8.85\times10^{-12})}(-\hat{\mathbf{j}}) + \frac{(0.5)(5\times10^{-6})}{2(8.85\times10^{-12})}(\hat{\mathbf{j}})$$

$$= \left[0.2 - 1 + 0.5\right]\frac{(5\times10^{-6})}{2(8.85\times10^{-12})}\hat{\mathbf{j}}$$

$$= -0.3\times10^{5}\ \text{N/C}$$

(d) What bottom cloud area charge density would suffice to initiate dielectric breakdown of the air, and precipitate a lightening strike?

So generally, the electric field in the air is:

$$\mathbf{E} = \mathbf{E}_{top\ cloud} + \mathbf{E}_{bottom\ cloud} + \mathbf{E}_{ground}$$

$$= \frac{\left|\sigma_{top\ cloud}\right|}{2\varepsilon_0}(-\hat{\mathbf{j}}) + \frac{\left|\sigma_{bottom\ cloud}\right|}{2\varepsilon_0}(\hat{\mathbf{j}}) + \frac{\left|\sigma_{ground}\right|}{2\varepsilon_0}(\hat{\mathbf{j}})$$

$$= \frac{(0.2)\left|\sigma_{bottom\ cloud}\right|}{2(8.85\times10^{-12})}(-\hat{\mathbf{j}}) + \frac{\left|\sigma_{bottom\ cloud}\right|}{2(8.85\times10^{-12})}(\hat{\mathbf{j}}) + \frac{(0.5)\left|\sigma_{bottom\ cloud}\right|}{2(8.85\times10^{-12})}(\hat{\mathbf{j}})$$

$$= \left[-0.2 + 1 + 0.5\right]\frac{\left|\sigma_{bottom\ cloud}\right|}{2(8.85\times10^{-12})}\hat{\mathbf{j}}$$

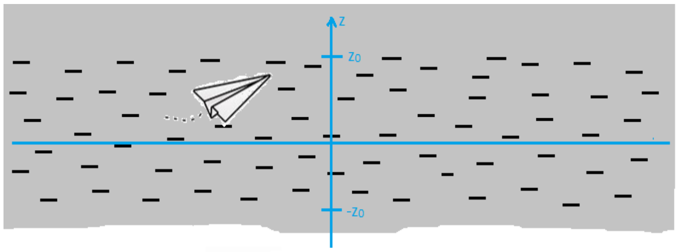
$$= 7.35\times10^{10}\left|\sigma_{bottom\ cloud}\right|\hat{\mathbf{j}}$$

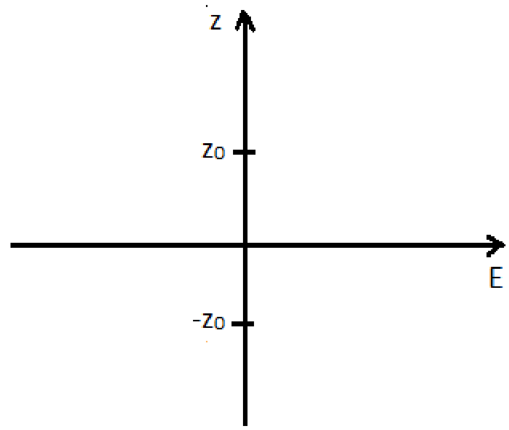
And we need,

$$3\times10^{6} = 7.35\times10^{10}\left|\sigma_{bottom\ cloud}\right|$$

$$\left|\sigma_{bottom\ cloud}\right| = \frac{3\times10^{6}}{7.35\times10^{10}} = 41\mu\text{C/m}^2$$

Problem 6. Let's reconsider the cloud. Suppose you're flying through the negatively charged bottom portion of it, which is centered about our coordinate system drawn below, and extends between $\pm z_0$ = $\pm$100m.

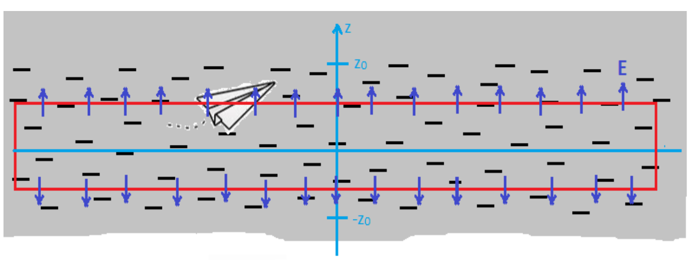




(a) Recalling the area charge density of the bottom cloud was -5μC/m², ascertain the volume charge density of the cloud.
So,

$$\sigma = \rho(2z_0) \rightarrow \rho = \frac{\sigma}{2z_0} = \frac{-5\,\mu\text{C/m}^2}{200} = -25\,\text{nC/m}^3$$

(b) Use Gauss's law to derive an expression for the magnitude and direction of the electric field the bottom cloud generates, as a function of z, and plot it below.
Electric field will be symmetric about the midline of the charge distribution (x-axis, we'll say). So we draw a Gaussian surface centered about origin, with height 2z along the z-axis, length ℓ along the x-axis, and width w along the y-axis (into the page).



Then according to Gauss's law, within the cloud, we have:

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\int_{top} \mathbf{E}\cdot d\mathbf{A} + \int_{bottom} \mathbf{E}\cdot d\mathbf{A} + \int_{left\,side} \mathbf{E}\cdot d\mathbf{A} + \int_{right\,side} \mathbf{E}\cdot d\mathbf{A} + \int_{front} \mathbf{E}\cdot d\mathbf{A} + \int_{back} \mathbf{E}\cdot d\mathbf{A} = \frac{\rho V}{\varepsilon_0}$$

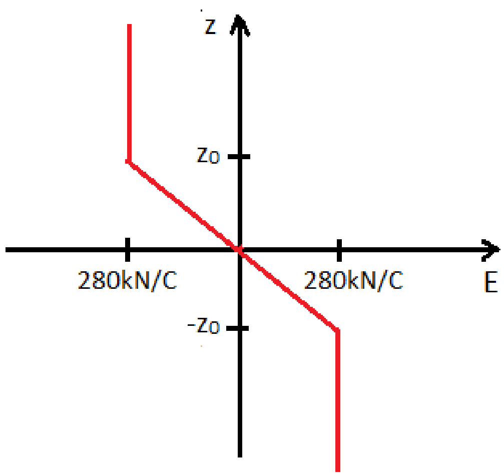
$$Elw + Elw + 0 + 0 + 0 + 0 = \frac{\rho\left[(2z)lw\right]}{\varepsilon_0}$$

$$E = \frac{\rho z}{\varepsilon_0}$$

Outside the cloud, we would have the usual result:

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{\rho(2z_0)}{2\varepsilon_0} = \frac{\rho z_0}{\varepsilon_0} = \frac{(25\times10^{-9})(100)}{8.85\times10^{-12}} = 2.8\times10^5 \text{ N/C}$$

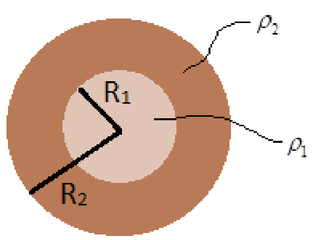
So if we plot the field, we'd have:

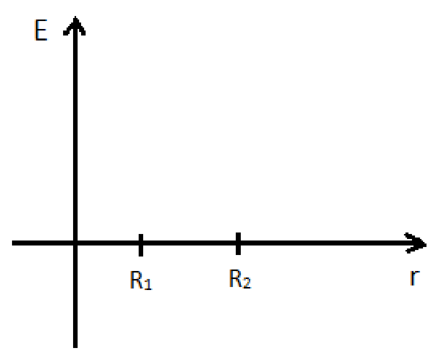


(c) At what z coordinate should you pilot your airplane to minimize the electrical interference the plane's circuity might experience as a result of the cloud's electric field (ignore the top portion and bottom portion of the cloud).

The minimum field is at z = 0, where E is in fact, 0. So you should fly in the middle of the cloud.

Problem 7. I'm gonna stick with the thunderstorm scenario. Maybe it's the weather. So recall how the negatively charged bottom part of the cloud inductively charged the ground (by repelling electrons from the surface of the ground, to deeper underground). As part of that scenario let's say that the branches of a tree have also, naturally, been charged in this fashion. Consider a typical cylindrical shaped branch whose cross-section is shown below. Say R_1 = 5cm, and R_2 = 10cm. And ρ_1 = 7pC/m^3, and ρ_2 = 3pC/m^3. Plot the electric field as a function of r (from the center of the branch). You may assume, for simplicity, that the infinite cylinder field formula is a good enough approximation here.





Inside the pith, we would have:

$$E = \frac{\lambda_{enclosed}}{2\pi\varepsilon_0 r} = \frac{\rho_1 \pi r^2}{2\pi\varepsilon_0 r} = \frac{\rho_1 r}{2\varepsilon_0} = \frac{(7\times10^{-12})(r)}{2(8.85\times10^{-12})} = 0.4r$$

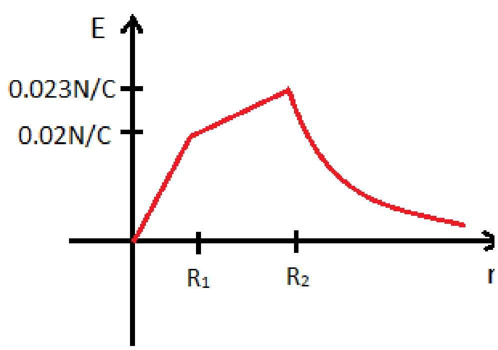
In the bark region we'd have:

$$E = \frac{\lambda_{enclosed}}{2\pi\varepsilon_0 r} = \frac{\rho_1 \pi r_1^2 + \rho_2(\pi r^2 - \pi r_1^2)}{2\pi\varepsilon_0 r}$$
$$= \frac{(\rho_1 - \rho_2) r_1^2}{2\varepsilon_0 r} + \frac{\rho_2 r}{2\varepsilon_0} = \frac{(7\times10^{-12} - 3\times10^{-12})(0.05)^2}{2(8.85\times10^{-12})r} + \frac{(3\times10^{-12})r}{2(8.85\times10^{-12})}$$
$$= \frac{5.65\times10^{-4}}{r} + 0.17r$$

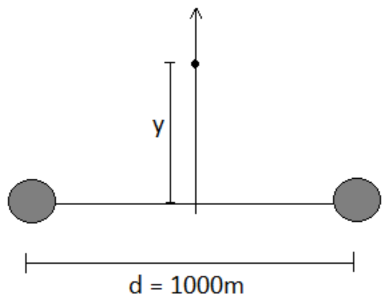
And outside the branch, we'd have:

$$E = \frac{\lambda_{enclosed}}{2\pi\varepsilon_0 r} = \frac{\rho_1 \pi r_1^2 + \rho_2(\pi r_2^2 - \pi r_1^2)}{2\pi\varepsilon_0 r} = \frac{(7\times10^{-12})(0.05)^2 + (3\times10^{-12})\left[0.10^2 - 0.05^2\right]}{2(8.85\times10^{-12})r}$$
$$= \frac{0.0023}{r}$$

Plotting this we'd get:



Problem 8. A top-down view of two 100m tall cell phone towers is shown. Suppose they are charged inductively (via the ubiquitous thunderstorm cloud) to 5mC each. Where along the y-axis is the net electric field strongest, and what is its value? (might want to recall from calculus how you find maximum values)



So electric field is given by:

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}) + \frac{\lambda}{2\pi\varepsilon_0 r}(-\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})$$

$$= \frac{\lambda}{\pi\varepsilon_0 r}\sin\theta\hat{\mathbf{j}}$$

$$= \frac{\lambda}{\pi\varepsilon_0 (y^2 + (d/2)^2)} \frac{y}{\sqrt{y^2 + (d/2)^2}}\hat{\mathbf{j}}$$

$$= \frac{\lambda y}{\pi\varepsilon_0 \left[y^2 + (d/2)^2\right]^{3/2}}\hat{\mathbf{j}}$$

Want to know where this is maximum, so:

$$\frac{d\mathbf{E}}{dy} = 0$$

$$\frac{\lambda}{\pi\varepsilon_0} \frac{(1)\left[y^2 + (d/2)^2\right]^{3/2} - (y)\frac{3}{2}\left[y^2 + (d/2)^2\right]^{1/2}(2y)}{\left[y^2 + (d/2)^2\right]^2} = 0$$

$$(1)\left[y^2 + (d/2)^2\right]^{3/2} - (y)\frac{3}{2}\left[y^2 + (d/2)^2\right]^{1/2}(2y) = 0$$

$$(1)\left[y^2 + (d/2)^2\right] - (y)\frac{3}{2}(2y) = 0$$

$$-2y^2 + (d/2)^2 = 0$$

$$y = \frac{d}{\sqrt{8}} = \frac{1000}{\sqrt{8}} = 353\text{m}$$

And then we plug this point into the E expression:

$$\mathbf{E} = \frac{\lambda y}{\pi\varepsilon_0\left[y^2 + (d/2)^2\right]^{3/2}}\hat{\mathbf{j}} \qquad \lambda = Q/L = 5\text{mC}/100\text{m} = 5\times10^{-5}\ \text{C/m}$$

$$= \frac{(5\times10^{-5})(353)}{\pi(8.85\times10^{-12})\left[(353)^2 + (500)^2\right]^{3/2}}\hat{\mathbf{j}}$$

$$= 2.77\ \text{N/C}\ \hat{\mathbf{j}}$$